

## QUESTION 2:

- (a) Use the Lagrange Interpolation to find the coefficients of the polynomial which passes through the four points **(0,0), (1,1), (2,0), (3,0)**. (45%)
- (b) Given an interval **[a,b]**, write down the formulas for finding the two interior points for the Golden Section method. What are the values of those interior points if **a=1** and **b=2**? Explain the reasons of why we use the Golden Section Method for interval reduction. (55%)

## SOLUTION

(a) By using the Lagrange Interpolation, the polynomial will be

$$P(x) = 0 + \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} + 0 + 0 = \frac{1}{2}x^3 - \frac{5}{2}x^2 + 3x,$$

i.e. the coefficients of the third degree polynomial are: 1/2, -5/2, 3, and 0.

(b) Given **a, b**, and the golden section number  $\Phi = 1.618\dots$ , the formulas for finding the two interior points **X2** and **X3** are

$$X2 = \Phi^{-1} a + (1 - \Phi^{-1}) b, \text{ and}$$

$$X3 = \Phi^{-1} b + (1 - \Phi^{-1}) a.$$

For **a = 1** and **b = 2**, we have:

$$X2 = \Phi^{-1} + 2(1 - \Phi^{-1}) = 2 - \Phi^{-1} = 1.382\dots, \text{ and}$$

$$X3 = 2\Phi^{-1} + (1 - \Phi^{-1}) = 2\Phi^{-1} + 1 - \Phi^{-1} = 1 + \Phi^{-1} = \Phi = 1.618\dots$$

The reason the Golden Section Method is so useful, is because it uses the Golden Ratio number to create two internal points with two very important properties:

1. The new line is a part of the original line, by keeping the two interior points and only one boundary point of the original line. By knowing those three points, we use the formulas above to calculate the new interior point.
2. The new line analogies between the boundary points and the two internal points are exactly the same with the original line.

It has been proved that the only number with this property is the Golden Ratio number, and each step/iteration (i.e. interval reduction) of our searching method should continue towards either left or right, depending on our specifications.

**Note:** The above answers (especially for part (b)) are only a **very detailed guide**. Brief and succinct answers should also be regarded as correct.