

Modelling Complex Systems

Mean Field Type Models

Exercises

Exercise 1: In a careful experimental study of the dynamics of populations of the metazoan *Daphnia magna*, it was found that the observations did not agree with the predictions of the logistic model. Using the mass M of the population as a measure of its size, he proposed the model:

$$\dot{M} = rM \left(\frac{K - M}{K + \alpha M} \right)$$

where r , K and α are positive constants.

Find the equilibrium points and determine their stabilities.

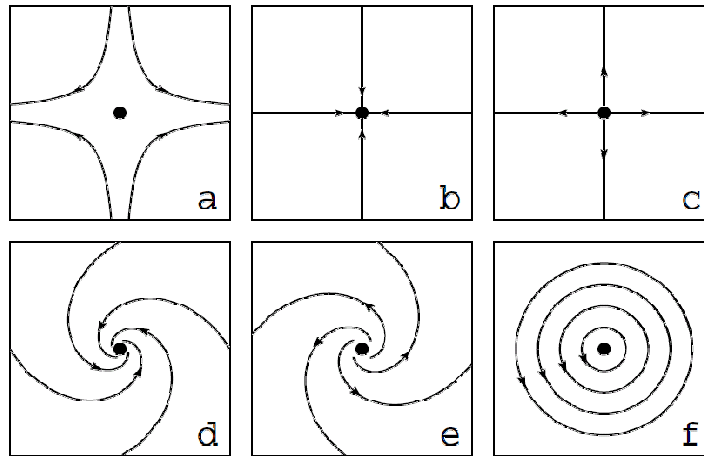
Exercise 2: In order to develop a strategy for harvesting a renewable resource, say fish, consider the equation:

$$\dot{N} = rN \left(1 - \frac{N}{K} \right) - H(N),$$

which is the usual logistic population model with an increase of mortality rate as a result of harvesting. $H(N)$ represents the harvesting yield per unit time.

- (1) Assuming $H(N) = CN$, where C is the intrinsic catch rate, find the equilibrium population N^* , and determine the maximum yield.
- (2) If, as an alternative strategy, we consider harvesting with a constant yield $H(N) = H_0$, determine the stable equilibrium point, and show that when H_0 approaches $\frac{1}{4}rK$ from below, there is a risk for the harvested species to become extinct.

Exercise 3: Discuss the stability of the following 2D systems represented by their phase portrait diagrams:



Exercise 4: An acceptable discrete one-population model of sexually reproducing organisms should be such that, if the population density is small, the organisms are sparsely distributed in their habitat, resulting in a low mating rate and a density-dependent growth rate less than 1. This effect is named after Allee. If the population density is large, intraspecific competition is strong, and it is reasonable to assume that, in this case, the density-dependent growth rate has to be less than 1. The model:

$$n_{t+1} = rn_t^x(1 - n_t)$$

where n_t is the population density at time t , and r and x are positive constants, is a generalization of the logistic model.

- (1) Assume $x > 1$, Under which condition(s) does the system exhibit a positive growth?
- (2) Assume $x = 2$, find the equilibrium points and determine their stabilities.

Consider the model:

$$n_{t+1} = rn_t^x(1 - n_t) - cn_t$$

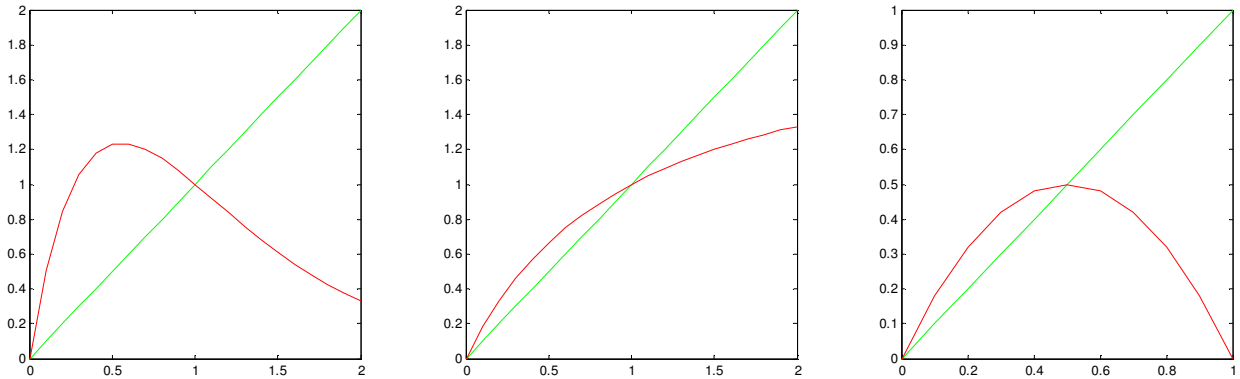
of harvesting a species described by the model above. The constant c is the intrinsic catching rate.

- (1) Show that, for a fixed value of r greater than a critical value r_c , the system undergoes a saddle-node bifurcation at a point (r^*, c^*)
- (2) What does this bifurcation imply from an ecological point of view?

Exercise 5: Determine the first period-doubling bifurcations of the one-parameter map:

$$n_{t+1} = -rn_t \log(n_t)$$

Exercise 6: discuss the stability of the maps below, plotted with a straight line with slope one and passing by the origin.



Exercise 7: discuss the stability of the map which the second iterate is plotted below for two values of the parameter r , with a straight line with slope one and passing by the origin.

