

## CITS4241 Visualisation Lecture 16

### Unstructured Point Sets: Dividing $k$ -dimensional space among $M$ data points

## Unstructured Points

- **Unstructured points are at irregular positions in 3D space, e.g.**
  - 'Slices' taken at irregular depths
  - Temperature distribution from thermocouples placed around a room
  - Weather stations distributed throughout some geographic region
  - ...
- **No topological structure to which we can apply our visualisation techniques!**
- **Common approaches**
  - Sample using a structured point set
    - **Splatting**
    - **Interpolation**
  - Create  $n$ -dimensional triangulations

## Splatting

- **Sample using a structured (regular) point set**
    - Create special influence or **splatting functions**
  - **Splatting functions**
    - Distribute data values over a surrounding volume
    - Gaussians common
- $$SF(x, y, z) = s \exp(-f(r/R)^2)$$
- Defines a region in which a value contributes to the value a distance  $r$  away from it
  - $s$  - scale factor (maximum value at  $r = 0$ )
  - $f$  - decay factor,  $f \geq 0$
  - $R$  - radius of influence

## Interpolation

- **Shepard's method**
  - is an inverse distance weighted interpolation technique
  - Given a set of  $n$  points,  $p_i = (x_i, y_i, z_i)$  and values  $F_i = F(p_i)$ , the interpolation function can be written as

$$F(p) = \frac{\sum_{i=1}^n \frac{F_i}{|p - p_i|^2}}{\sum_{i=1}^n \frac{1}{|p - p_i|^2}}$$

- + Easy to implement, but
- derivatives are zero at each  $p_i$  → limits its usefulness for most practical applications

## Triangulation

- Builds topology directly from unstructured points
- We have
  - Point set,  $P = (p_1, p_2, p_3, \dots, p_n)$
- A  $n$ -dimensional triangulation is a collection of  $n$ -dimensional simplices whose defining points are in the set  $P$
- Simplicies
  - do not intersect each other
  - share only edges or faces
- Delaunay Triangulation
  - Important form of triangulation

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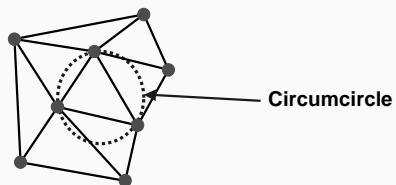
## What's a simplex?

- A simplex of dimension  $n$  is
  - a convex region defined by a set of  $n+1$  points in general position
- Simplicies

Dimension	Shape
0	Vertex
1	Line segment
2	Triangle
3	tetrahedron

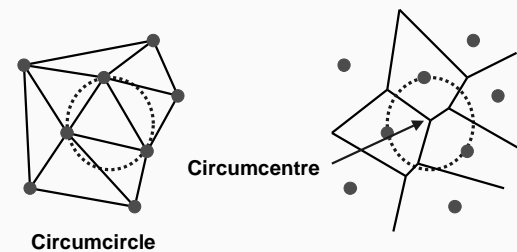
## Triangulation

- Delaunay Triangulation
  - Important form of triangulation
  - Circumsphere contains only the  $n+1$  defining points of  $P$
  - In 2D (i.e.,  $n = 2$ )



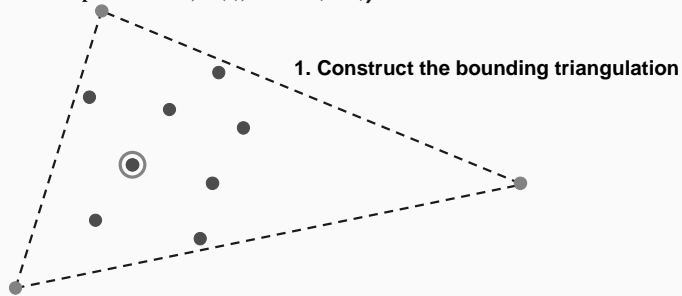
## Triangulation

- Dirichlet or Voronoi Tessellation
  - Dual of Delaunay Triangulation
  - Defines regions which are closer to one point than any other
  - Intersections are centres of circumcircles



## Delaunay Triangulation - Generation

- **Watson** (D F Watson, "Computing the n-dimensional Delaunay Tesselation ..", The Computer Journal, 24(2), 167-172 (1981)) **and** **Bowyer** (A Bowyer, "Computing Dirichlet Tesselations ", The Computer Journal, 24(2), 162-166 (1981))

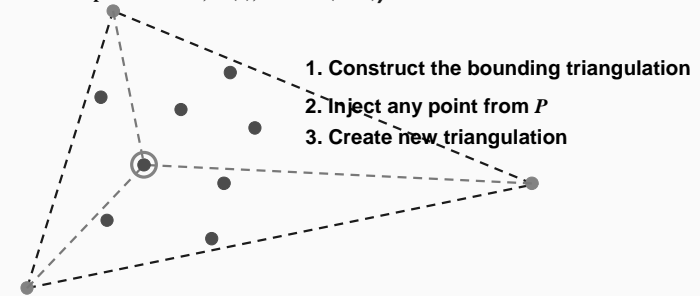


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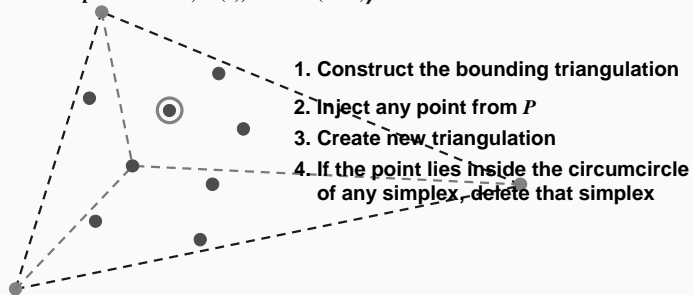


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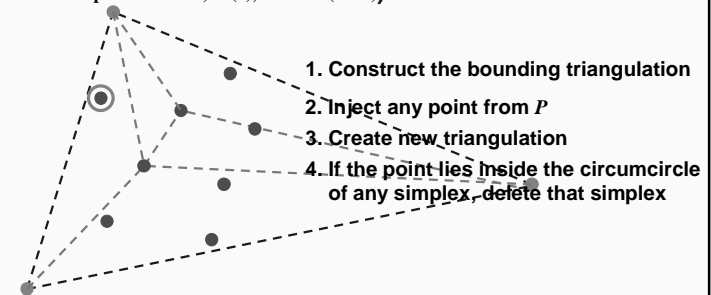


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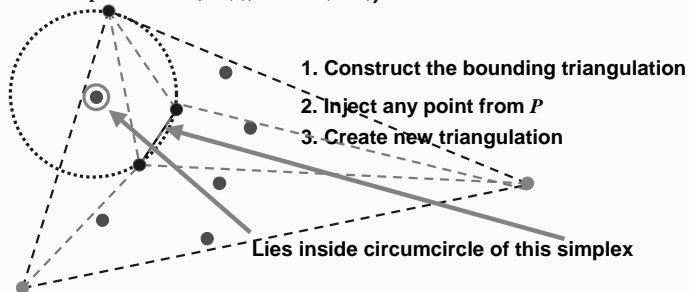


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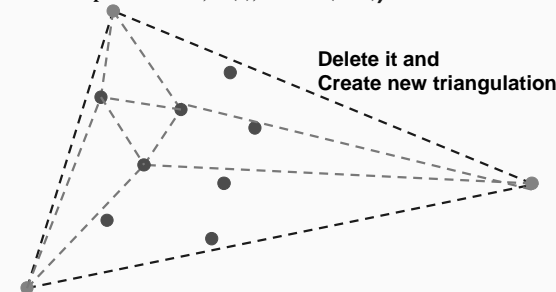
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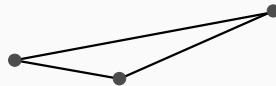
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## Difficulties

- **Degeneracies**
  - Points lying at vertices of a square
  - Each triangulation is equivalent
- **Numerical problems**
  - Particularly with simplices of high aspect ratio



## Further reading

- **Dividing  $k$ -dimensional space among  $M$  data points**
  - Schroeder et al, "The Visualization Toolkit – An Object Oriented Approach to 3D Graphics", 1998, Chapter 9, pages 394 – 400