

Artificial Intelligence

Topic 12

Logical Inference

Reading: Russell and Norvig, Chapter 7, Section 5
Extra reading: Russell and Norvig, Sections 8.1, 8.2, 9.4 (not examinable)

Outline

- ◇ Inference systems
- ◇ Soundness and Completeness
- ◇ Proof methods
 - normal forms
 - forward chaining
 - backward chaining
 - resolution
- ◇ More logics
- ◇ Logic Programming

From Entailment to Inference

- We have answered what it means to say α follows from a knowledge base KB :
 - $KB \models \alpha$
- We have seen that this can be determined semantically by *model checking* or by *truth table enumeration*
 - 2^n models or rows for n symbols
- Is there a better way?
 - can we do it from syntax alone?
 - can we automate it?
 - can we even turn it into a programming language?

Inference Systems

Inference system - set of *rules* for deriving new sentences from existing ones

- AKA **Proof System, Derivation System, Theorem-Proving System**
- rules operate directly on syntax

Example:

$P_1 =$ Socrates is a man

$P_2 =$ Socrates is mortal

$P_1 \Rightarrow P_2$ (If Socrates is a man, then Socrates is mortal)

Assume $KB = \{P_1, P_1 \Rightarrow P_2\}$.

We know $KB \models P_2$. (check)

What about inference rules?

Inference Systems

Modus Ponens

$$\frac{\alpha \quad \alpha \Rightarrow \beta}{\beta}$$

pattern matching — from sentences that match α and $\alpha \Rightarrow \beta$, generate a new sentence that matches β

More examples...

And Elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

Or Introduction

$$\frac{\alpha}{?}$$

Inference Systems

Modus Tolens

$$\frac{\neg\beta \quad \alpha \Rightarrow \beta}{\neg\alpha}$$

An inference system may contain one or more inference rules.

Notation:

$KB \vdash \alpha$ = sentence α can be derived from KB using the rules of the inference system

Soundness and Completeness

In fact we could make up any inference rule we like. How about:

And Introduction

$$\frac{\alpha}{\alpha \wedge \beta}?$$

Why wouldn't we want this rule in our inference system?

Soundness and Completeness

We only want rules that “correspond” to logical consequences. Formally...

Soundness: an inference system is *sound* if

whenever $KB \vdash \alpha$, it is also true that $KB \models \alpha$

That is, it *only* allows you to generate logical consequences.

Completeness: an inference system is *complete* if

whenever $KB \models \alpha$, it is also true that $KB \vdash \alpha$

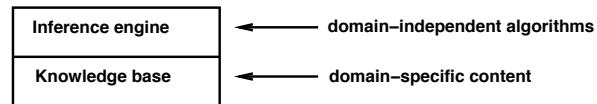
That is, it allows you to generate *all* logical consequences.

(Which do you think is worse, sound but not complete, or complete but not sound?)

Ideally we would like to use an inference system that is both sound and complete.

Review of Progress

Recall our logical agent:



- **Knowledge base** = set of **sentences** in a **formal** language ✓
- **TELL** it what it needs to know ✓
($KB \leftarrow KB \cup \{\alpha\}$)
- **ASK** — answers should follow from the KB ✓?
($KB \vdash \alpha$)

A sound and complete inference system means that if α follows from KB then there is a sequence of rule applications that allow you to generate α starting with KB — but it doesn't tell you *how* to get there!

Proof Methods

Consequences of KB are a haystack; α is a needle.
Entailment = needle in haystack; inference = finding it



Proof Methods

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- **Proof** = a sequence of inference rule applications
Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a **normal form**

Examples

- forward and backward chaining (Horn form)
- resolution (conjunctive normal form)

Horn Form

Horn Form (restricted)

KB = **conjunction** of **Horn clauses**

Horn clause =

- ◇ *proposition symbol* or
- ◇ (*conjunction of symbols* \Rightarrow *symbol*)

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

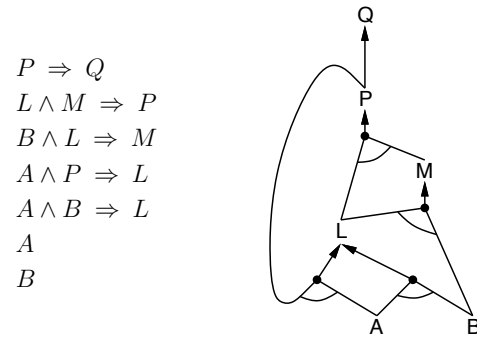
Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \dots, \alpha_n, \quad \alpha_1 \wedge \dots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

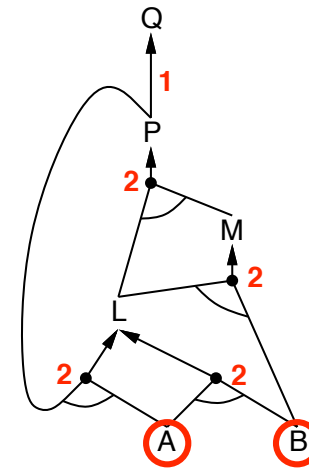
Can be used with **forward chaining** or **backward chaining**.
These algorithms are very natural and run in linear time.

Forward chaining

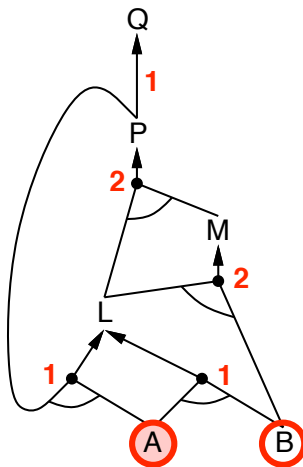
Idea: systematically iterate through knowledge base,
fire any rule whose premises are satisfied in the *KB*,
add its conclusion to the *KB*, until query is found



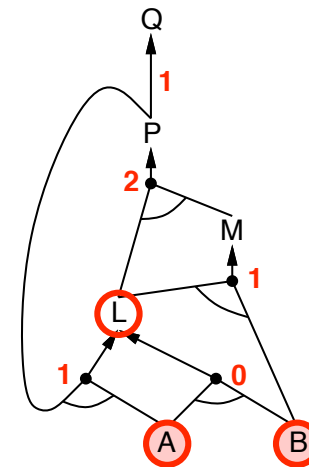
Forward chaining example



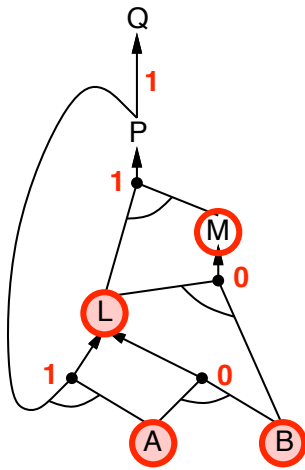
Forward chaining example



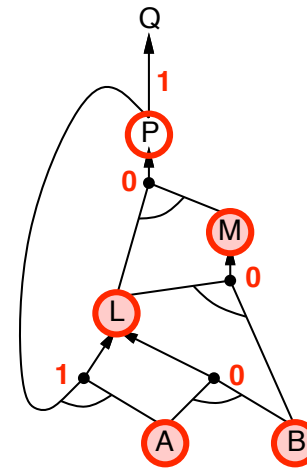
Forward chaining example



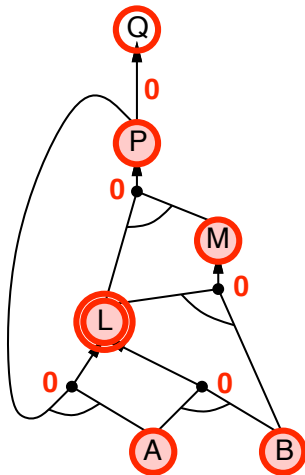
Forward chaining example



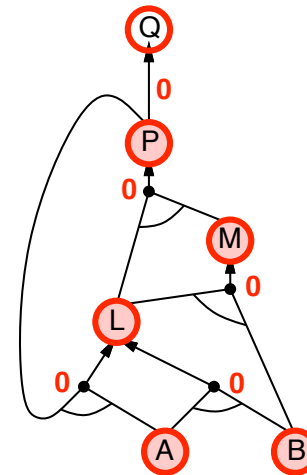
Forward chaining example



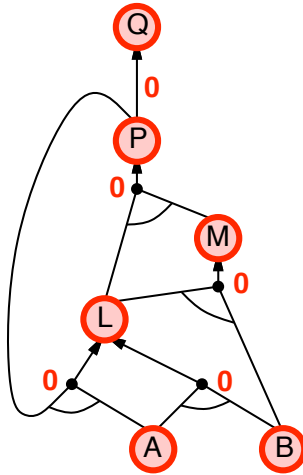
Forward chaining example



Forward chaining example



Forward chaining example



Backward chaining

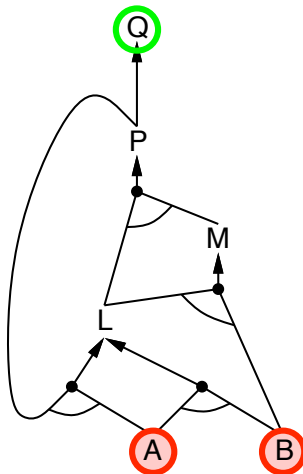
Idea: work backwards from the query q :
to prove q by BC,
check if q is known already, or
prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

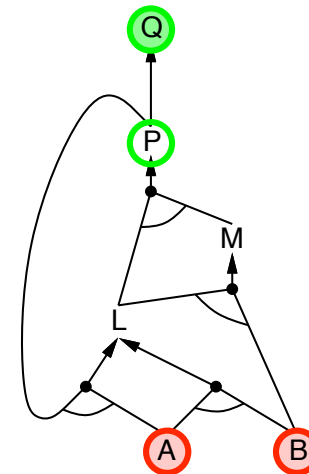
Avoid repeated work: check if new subgoal

- 1) has already been proved true, or
- 2) has already failed

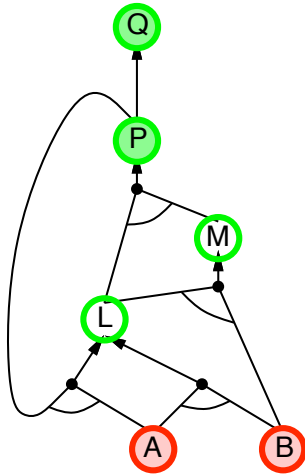
Backward chaining example



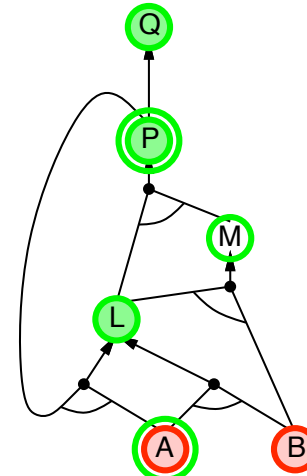
Backward chaining example



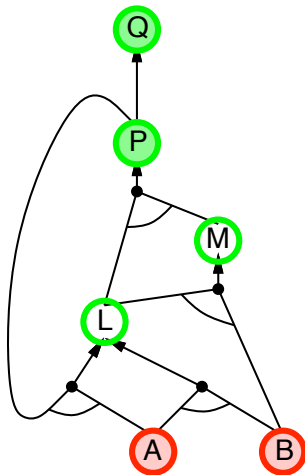
Backward chaining example



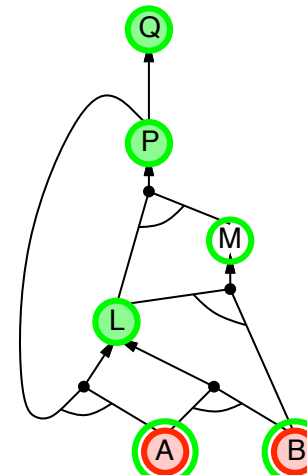
Backward chaining example



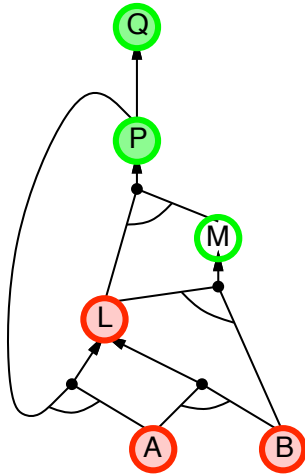
Backward chaining example



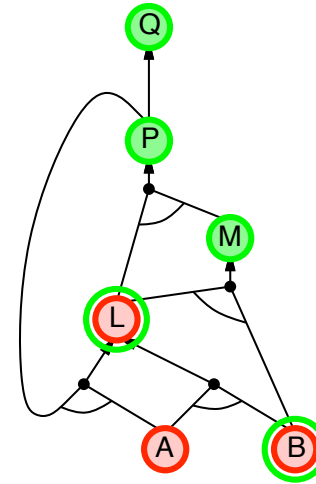
Backward chaining example



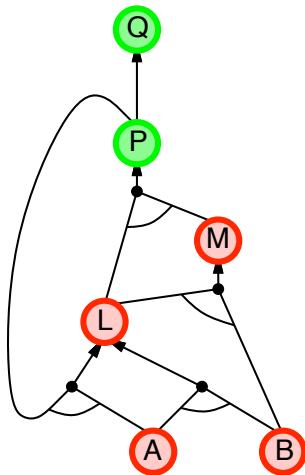
Backward chaining example



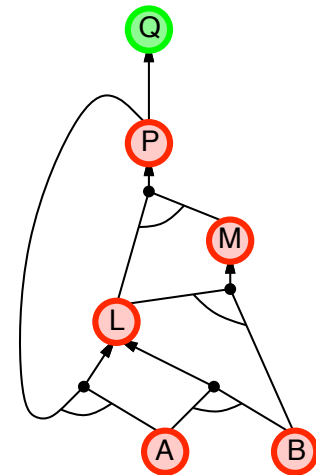
Backward chaining example



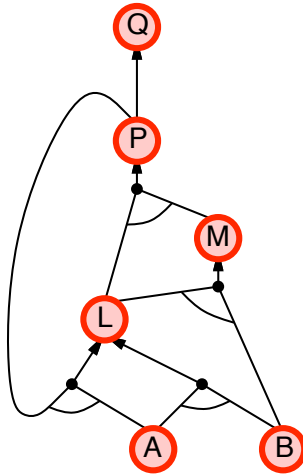
Backward chaining example



Backward chaining example



Backward chaining example



Forward vs. backward chaining

FC is **data-driven**, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is **goal-driven**, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be **much less** than linear in size of KB

Resolution

Conjunctive Normal Form (CNF—universal)
conjunction of **disjunctions** of **literals**
clauses

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

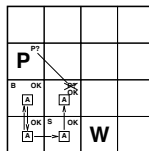
Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \dots \vee \ell_k, \quad m_1 \vee \dots \vee m_n}{\ell_1 \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \quad \neg P_{2,2}}{P_{1,3}}$$

Resolution is sound and complete for propositional logic



Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move \neg inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law (\vee over \wedge) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Resolution algorithm

“Proof by contradiction”

Based on the fact that $KB \models \alpha$ iff $KB \cup \{\neg\alpha\}$ is unsatisfiable (prove!)

Unsatisfiability in CNF is indicated by the **empty clause**

(Consider the number of models satisfying

$$(A \vee B \vee C \vee \dots)$$

$$(A \vee B \vee C)$$

$$(A \vee B)$$

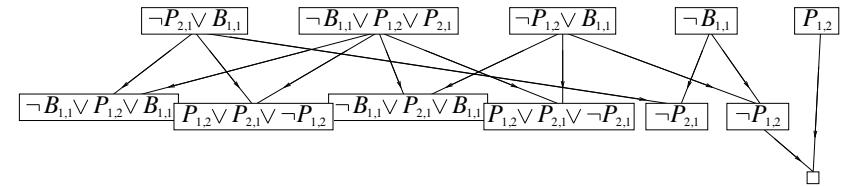
$$(A)$$

$$()$$

Vacuously false)

Resolution example

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \quad \alpha = \neg P_{1,2}$$



More Logics

Although propositional logic is computationally attractive, it lacks expressive power in practice.

eg. How would you say “All men are mortal” or “All squares adjacent to a pit have a breeze”?

There are many other logics that extend propositional logic, eg:

- first-order logic introduces objects, functions, relations, variables, quantifiers (*for all, there exists*)
- higher order logics allow the logic to refer to its own constructs
- temporal logics introduce specific structures to represent time steps
- modal logics introduce possibility and necessity
- probabilistic logics introduce the probability a statement is true
- fuzzy logics introduce a degree of membership to a class

Logic Programming

PROLOG

- most common logic programming language (recall Japanese 5th gen.)
- resolution inference rule
- first-order logic
- goal-directed, depth-first search (no occurs check)

Others include:

- GÖDEL — second-order logic
- GOLOG — agent programming

Inductive Logic Programming (ILP) — combining inductive learning with logic programming

Summary

Logical agents apply **inference** to a **knowledge base**
to derive new information and make decisions

Basic concepts of logic:

- **syntax**: formal structure of **sentences**
- **semantics**: **truth** of sentences wrt **models**
- **entailment**: necessary truth of one sentence given another
- **inference**: deriving sentences from other sentences
- **soundness**: derivations produce only entailed sentences
- **completeness**: derivations can produce all entailed sentences

Forward, backward chaining are linear-time, complete for Horn clauses

Resolution is complete for propositional logic

Resolution is the basis of the Prolog programming language

Uses first-order logic — more expressive power