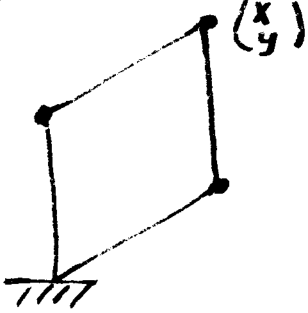
REDUNDANT ROBOTS.

Up until now we have only considered kinematically non-redundant robots: that is, the number of degrees of freedom of the mechanism has been exactly equal to the number of variables we wanted to specify.

A kinematically non redundant arm is also called a <u>minimum dexterity</u> arm. Nevertheless such an arm does not necessarily have a <u>unique</u> configuration satisfying the constraints

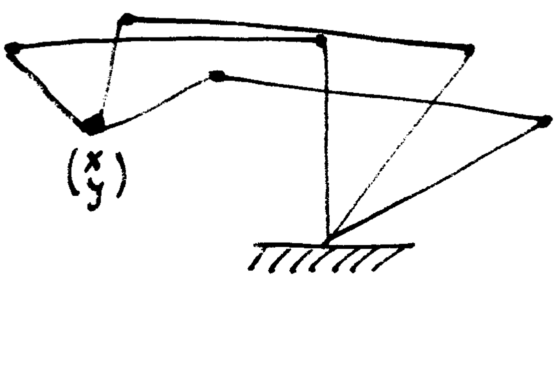
e.g.



"elbow up" and "elbow down" solutions satisfy position constraint $\binom{x}{y}$.

A <u>kinematically redundant</u> arm is one that has more degrees of freedom in its mechanism than are required for the task.

In such cases there is an infinite number of arm configurations that satisfy any desired position and orientation of the end effector.



Although the solution of the kinematic equation is now more complex, redundant robots have the advantage of increased effective working area and increased dexterity of the arm, e.g., the robot can reach into confined spaces or around corners.

For the motion rate control equations the constraint vector $\dot{\underset{\sim}{x}}$ has a smaller dimension than the actuator velocity vector $\dot{\underset{\sim}{\theta}}$. We thus have

$$(1) \qquad \dot{x}_{m\times 1} = J_{m\times n}\,\dot{\theta}_{n\times 1} \qquad \text{with } n > m$$

The system is underconstrained.

Such a system of equations can still be solved by choosing some performance constraints such as minimising the system's kinetic energy or minimising $\|\dot{\theta}\|$.

This gives a solution of the form

$$(2) \qquad \dot{\underset{\sim}{\theta}} = G\,\dot{\underset{\sim}{x}}$$

where G is some <u>generalized inverse</u> of J, that is, any matrix satisfying

$$JGJ = J$$

(If such a G exists then clearly (2) is a solution of (1))

Theorem: A generalized inverse always exists.

<u>Proof:</u> Given any $J_{m \times n}$, let $r = \text{rank } J$ where $\text{rank } J < m < n$. Then there exist nonsingular matrices $P_{m \times m}$ and $Q_{n \times n}$ such that

$$PJQ = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}_{m \times n}$$

where I_r is the $r \times r$ identity matrix.

Letting

$$G = Q \begin{pmatrix} I_r & U \\ V & W \end{pmatrix}_{n \times m} P ,$$

where U, V and W are arbitrary, we have a generalized inverse, since

$$JGJ = J\,Q \begin{pmatrix} I_r & U \\ V & W \end{pmatrix} P\,J$$

$$= P^{-1} PJQ \begin{pmatrix} I_r & U \\ V & W \end{pmatrix} PJQ\,Q^{-1}$$

$$= P^{-1} \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} I_r & U \\ V & W \end{pmatrix} \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} Q^{-1}$$

$$= P^{-1} \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} Q^{-1}$$

$$= J.$$