CITS7211 Modelling Complex Systems
Project 2: Modelling infectious Disease Spread
Due: 5pm Friday, 5th June

Individual reports are required. The report and the codes must be submitted via cssubmit by the due date and time.

This project is worth 20% of your final grade for the unit.

Problem description:

Part A:

In an SIR model for the spread of an infectious disease, based on disease status, the individuals are divided into three disjoint groups:

- **Susceptible** individuals are capable of contracting the disease and becoming infective.
- **Infective** individuals are capable of transmitting the disease to susceptible individuals.
- **Removed** individuals have had the disease and are dead, have recovered and are permanently immune, or are isolated until recovery and permanent immunity occur.

Let S_t, I_t, and R_t denote the densities of individuals belonging to the three different groups defined above. If we assume that the interactions responsible for the spread of the disease have a very long range (*i.e.*, each susceptible individual can be equally infected by any infective individual), it can be shown that the spread of the infectious disease can be modeled by the recurrence:

$$\begin{cases} S_{t+1} + R_{t+1} + I_{t+1} = \rho \\ I_{t+1} = I_t + S_t\left(1 - e^{-iI_t}\right) - rI_t \\ R_{t+1} = R_t + rI_t \end{cases}$$

where i is a coefficient proportional to the probability for a susceptible individual to be infected, r is the probability for an infective individual to be removed, and ρ is the total initial density.

(A.1) Show that as time t goes to infinity, the limits S_∞, I_∞, and R_∞ exist. What is the value of I_∞?

(A.2) Discuss all possible evolutions of the model with regard to the model parameters and the initial conditions.

Part B:

Extend the previous one-population SIR epidemic model to describe the spread of a sexually transmitted disease among heterosexual individuals.

(B.1) Show that the limits $S_{m\infty}$, $I_{m\infty}$, $R_{m\infty}$, $S_{f\infty}$, $I_{f\infty}$, and $R_{f\infty}$, where the superscripts m and f refer, respectively, to the male and female populations, exist. What are the values of $I_{m\infty}$ and $I_{f\infty}$?

(B.2) According to the initial conditions and the model parameters, discuss the existence of epidemics in one or both populations.

Part C:

Consider a situation in which, after recovery, infected individuals again become susceptible to catch the disease (as, e.g., with common cold).

(C.1) Assuming, as in Part A, that each susceptible individual can be equally infected by any infective individual, derive the recurrence equations satisfied by the densities of susceptible and infected individuals denoted, respectively, by S_t and I_t.

(C.2) Show that this model exhibits a transcritical bifurcation between an endemic state (*i.e.*, a state in which the stationary density of infected individuals I_∞ is nonzero), and a disease-free state in which I_t is equal to zero.

Deliverable A – Codes: You should submit the codes used to simulate the models of the three parts A, B and C. **10 %**

Deliverable B – The Report: The report should be between 10 and 20 pages and should include the following sections:

- A summary, max 100 words. **5%**
- An Introduction. **5%**
- A bibliographical review of modeling diseases spread, **10%**
- Detailed discussions of the models in the three parts. Address the different points (A.1 to C2). Illustrations of the different cases. **55%=(A)15%+(B)25%+(C)15%**
- Limitations of the studied model. **10%**
- Conclusions. **5%**